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DETECTION OF GRAVITATIONAL COLLAPSE

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ABSTRACT

At least one kind of supernova is expected to emit a large flux of neutrinos and gravitational radiation because of the collapse of a core to form a neutron star. Such collapse events may in addition occur in the absence of any optical display. The corresponding neutrino bursts can be detected via Cerenkov events in the same water used in proton decay experiments. Dedicated equipment is under construction to detect the gravitational radiation. Events throughout the Galaxy could be detectable, but are expected only at intervals exceeding a decade. Nevertheless, the next event could come tomorrow, so every attempt should be made to make the monitoring for such events routine.

SUPERNOVAE AND GRAVITATIONAL COLLAPSE

Bursts of neutrinos and gravity waves are expected from the gravitational collapse of a stellar core to form a neutron star. Such collapse is expected for the cores of massive stars and is thought to be connected with the explosion mechanism of supernovae of Type II. On the other hand, some collapse events may give no burst of light, only neutrinos and gravity waves.

Type II supernovae are identified by their normal hydrogen-rich spectra. Studies of their spectra, light curves and their correlation with spiral arms all point to the precursor being a massive star. The rate of explosion of Type II supernovae in the Galaxy suggests that they come from stars of about 10 to 20 $M_\odot$. Theoretical studies of the evolution of stars of such mass show that the formation of an iron core of about 1.5 $M_\odot$ and its subsequent collapse is very likely[1]. (Severe internal rotation might alter this conclusion.) The rates of explosion of Type II supernovae are roughly equal to the rate of formation of pulsars, rotating magnetized neutron stars. Thus with canonical numbers there is some reason to think that the supernova explosion in a star of 10-20 $M_\odot$ is associated with gravitational collapse. This association with pulsars may be a bit simplistic, as will be argued below.

Above 20 $M_\odot$ some stars may make neutron stars. Others may undergo total collapse to give black holes. There is no present certainty as to which stars do which at any particular mass. The rate of events from such massive stars is very small, less than one per century in the Galaxy. There may be a different neutrino signature for the two events. A neutron star may have a rather long deleptonization phase in which the binding energy of the neutron star, $\sim 10^{53}$ ergs, is radiated. The neutrino burst from the formation of a black hole is likely to be truncated after $\sim$ 10 msec. Thus, the detection of one of these rare events would be very useful.

Type I Supernovae are hydrogen deficient and uncorrelated with the arms of spiral galaxies. They do seem to be correlated with

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regions of active star formation in small irregular galaxies. The precursor stars are presumably less massive than those of Type II supernovae, but the nature of these precursors is uncertain.

A currently popular model for Type I supernovae is one in which a thermonuclear explosion results in total disruption, and no collapse[2]. Such an explosion would produce neutrinos but they would be few in number and have an energy of order one MeV which would make them virtually impossible to detect.

There is some evidence in favor of the prediction that Type I events do not leave neutron stars. Observations with the Einstein Orbiting X-ray Observatory have failed to detect thermal X-ray emission from the hot surface of neutron stars which might be present in the putative Type I remnants of SN 1006, SN 1572, and SN 1604[3]. An independent constraint is the failure to see an X-ray synchrotron nebula in these remnants. This is an important constraint because such a nebula is not subject to pulsar beaming which could hide a radio pulsar. Such a nebula is obvious in the Crab Nebula and detected for the Vela supernova remnant as well as around some pulsars[4].

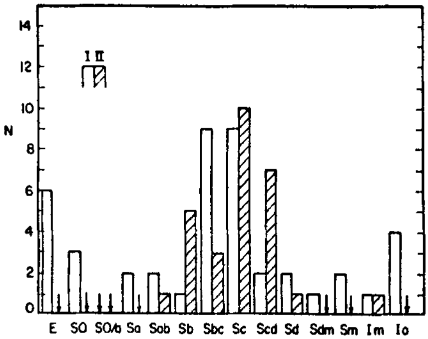
If the thermonuclear explosion models are correct, then Type I supernovae should be discounted when trying to estimate collapse rates from supernova rates. On the other hand, if collapse is involved and a neutrino or gravity wave burst could be detected, then the whole class of thermonuclear models could be discarded. A definite statement about the simple existence of a neutrino or gravity wave burst or lack thereof associated with a Type I event would be very useful.

EVENT RATES

The total rate of supernova events in the Galaxy is roughly one per 25 years[5]. This rate results from estimates based on historical rates corrected for Galactic extinction, or by interpolation from extragalactic rates in different types of galaxies. Pulsars are formed at a rate of about one per 35 $\pm$ 15 years[6]. There is no useful estimate of the rate of non-magnetic non-optical events.

The total supernova rate in the Galaxy will not be the relevant number to estimate gravitational collapse if Type I events do not produce collapse. Unfortunately, the ratio of Type I to Type II events in the Galaxy is very uncertain. Of the seven or so historical events, three are putatively of Type I. The light curves of SN 1572 and SN 1604 by Tycho and Kepler[7] are roughly more consistent with Type I than Type II. SN 1006 was very bright and so may have been a Type I which are typically about three times brighter than Type II. Assigning a type to SN 1054 is very difficult despite the historical record. It clearly left very different remnants, both compact and extended, than SN 1006, SN 1572 or SN 1604. Arguments that it may have been Type II depend on assigning the first Chinese record to maximum light. The Japanese, however, apparently saw it earlier, and the a priori chance that they saw it on the rapid rise is small. Without knowing the phase of the early observations, no useful constraint on the light curve can be assigned. Cas A may not have been observed at all and in any case was too dim to be either classical type of supernova. Thus the only identifiable supernovae in the Galaxy were Type I. None can be assigned as Type II.

Figure 1 - The number of supernovae definitely identified to be of Type I or Type II is presented as a function of the type of the host galaxy in the sample of Oemler and Tinsley[7].



In other galaxies the ratio of Type I to Type II varies with the type of galaxy as shown in Figure 1. No Type II are seen in gas poor elliptical galaxies nor in IO galaxies which are small irregular galaxies composed mostly of old stars but interspersed with patches of active star formation. This suggests that Type I events are and are not associated with moderately massive stars, adding to the confusion concerning their precursors. Spiral galaxies are classified by the degree of tightness with which the arms are wrapped, tightly for Sa, loosely for Sc. Sc galaxies are prodigous supernova producers and the numbers of Type I and Type II are roughly equal. In the sample of Sa and Sb galaxies the ratio is not at all clear. The probability of seeing one Type I and five Type II in Sb galaxies when the true rates are equal is only a few percent. The ratio of Type I to Type II may vary rapidly with galactic type. The Galaxy is roughly an Sbc, right in the range of high uncertainty.

The standard assumption is that the rates of Type I and Type II are equal. Then Type II occur at roughly one per 50 years. This is roughly comparable to the pulsar rate as mentioned previously.

On the other hand, the historical rates may show a bias toward Type I. In addition, the dearth of pulsars is not confined to the three historical remnants associated with Type I. The search for X-ray synchrotron nebulae has revealed nothing detectable in the majority of older supernova remnants surveyed. If these events made pulsars they should be detectable because pulsars live $\sim 10^7$ years versus $\sim 10^5$ years for the remnants. If there is evidence for pulsars in only roughly 20 percent of the extended remnants, one might argue that only 20 percent of the supernovae were Type II, given the uncertainty in the Type I/Type II ratio. Then the rate of formation of pulsars in supernovae would be about one in 125 years. This can not be the only pulsar production mechanism since such a rate disagrees seriously with the observed pulsar formation rate.

The fact that most remnants show no evidence for pulsars implies that there are ways to make pulsars without producing an extended remnant. Therefore, one should scale estimates of collapse rates with the supernova rates only with great caution.

If there are non-magnetic collapse events in Type I and Type II supernovae or events without any optical display, then the rate of collapse would be higher than either the pulsar rate or the supernova rates imply. A very optimistic estimate for the rate of collapse in the Galaxy would be about one every ten years. How does this compare with the death rate expected from the known number of stars and their astrophysically calculated evolution rates? Comparable if collapse occurs in all stars with $M > 4\ M_{\odot}$. A $4\ M_{\odot}$ limit is a low, but not impossible number. Collapse may be triggered in old white dwarfs by mass transfer in binary systems or slow changes in composition at a rate unrelated to the current star formation rate so no specific mass limit can be assigned. A rather pessimistic limit to the collapse rate would be the upper limit for pulsars, about 1 per 50 years, corresponding to all stars above about $10\ M_{\odot}$.

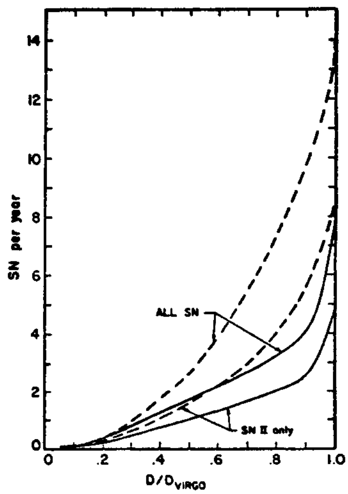


Figure 2 - The rate at which supernovae are expected to occur in external galaxies is given as a function of the fractional distance from our Galaxy to the center of the nearby rich Virgo cluster of galaxies. Estimates for this distance range from 15 to 25 Megaparsecs. The solid lines indicate the rate of supernova events between r and r + dr. The dashed lines give the integrated rate out to the distance r. In each case, the upper curve gives the expected rate for all supernova types, and the lower curve gives the rate for Type II supernovae only.

Extragalactic supernova rates are much higher at sufficient distance. Figure 2 shows the rates of all supernovae, and the rates of Type II only, in successive spherical shells out to the Virgo cluster. Also shown is the integrated rate. These numbers are based on an analysis by Tammann[8], but with care taken not to apply correction factors intended for spiral galaxies and for Type II to all galaxy types and all supernovae. The distance to Virgo and intermediate points is as uncertain as the Hubble constant and the age of the Universe. Estimates for the distance to Virgo center therefore range from 15 to 25 Mpc. The rates for supernovae approach one per year for distances of ~ 3-5 Mpc. Unfortunately, the neutrino and gravity wave signals from such a distance will remain impractically small for the foreseeable future.

DETECTING THE NEUTRINO BURST FROM COLLAPSE

The solar neutrino experiment operated by Ray Davis and his colleagues (most recently reviewed in these proceedings) could detect a nearby collapse. For larger, more sensitive experiments, simple water may be the most effective neutrino detector. The detection mechanisms are

$\bar{\nu}_e$ interacts with a proton ($n = 2$ per H_2O)

with a cross section

$$\sigma(\bar{\nu}_e, p) = 8.5 \times 10^{-42}\ \mathrm{cm}^2\ (\varepsilon_{\bar{\nu}_e}/10\ \mathrm{MeV})^2,$$

and

ν_e interacts with an electron ($n = 10$ per H_2O)

with a cross section

$$\sigma(\nu_e, e^-) = 1.7 \times 10^{-43}\ \mathrm{cm}^2\ (\varepsilon_{\nu_e}/10\ \mathrm{MeV}).$$

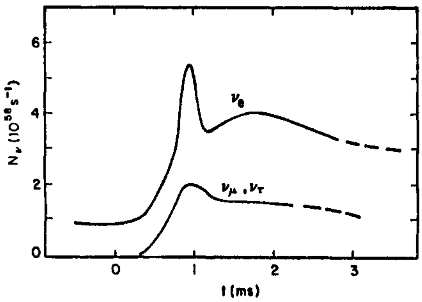
Any interaction with oxygen is negligible by comparison. For each mechanism of detection one counts on measuring the number of interactions N_d that take place in a mass of water M in the total time scale of interest. That is governed by the time integral, F(particles/cm^2), of the flux at the earth of the neutrinos from the star,

$$F = 8.3 \times 10^9\ \mathrm{cm}^{-2}\ N_{56}\ (D/10\ \mathrm{kpc})^2.$$

Here $N_{56} = N/10^{56}$ is the neutrino emission from the star totaled over the time of interest and D is the distance from the star. The expected number of events in the detector via a given mechanism is

$$N_d = nN_0F\ M/\mu$$

Current calculations of gravitational collapse differ in detail but agree in general outline. After neutrinos are trapped at about 10^{12} g cm^{-3} the central portions of the star collapse in a low entropy, homologous ($v \propto r$) fashion. The collapse of this inner core halts when it reaches nuclear density and the equation of state stiffens. A shock forms as material rains in on the halted inner core.



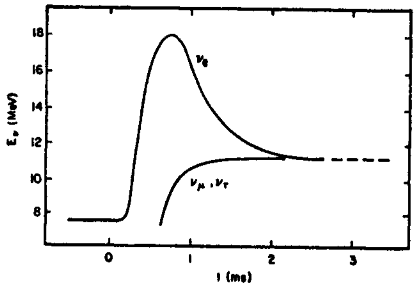


Figure 3 - The rate of emission of neutrinos during the gravitational collapse of a stellar core is given as a function of the time since the formation of the shock when the core bounces at nuclear densities. The upper curve presents the rate of electron neutrinos which result from electron capture. The lower curve gives the rate of other neutrinos from thermal processes. The curves shown are a somewhat schematic representation of the results of Mazurek, Cooperstein and Kahana[9] and those of Van Riper[10].

Figure 4 - Representative energies are given for the neutrinos emitted during gravitational collapse.

Neutrino emission begins with electron capture in the collapsing material in the regions beyond the neutrino trapping radius. A short intense relatively energetic burst accompanies the arrival of the shock at the "neutrinosphere", $\tau_{\nu_e} \sim 1$ as shown in Figures 3 and 4. Energy can rise to ~ 20 MeV per neutrino for ~ 1 msec. A plateau of ~ 10 msec then follows in which electron capture proceeds in the post-shock material. Energy drops to ~ 10 MeV, but the pair formation process can begin to provide a flux of $\bar{\nu}_e$ comparable to ν_e.

Over longer timescales the situation is uncertain. A period of "deleptonization" ensues in which the binding energy of the neutron star is emitted as neutrinos. This results in the release of $\sim 10^{53}$ ergs worth of neutrinos but the timescale and energy spectrum are uncertain, depending, for instance, on whether the core is quasi-static or subject to strong hydrodynamic motions. The neutrino energy is also uncertain.

The original burst is predicted to give $\sim 10^{56}$ ν_e of $\varepsilon_{\nu_e} \sim 10 - 20$ MeV over a period of ~ 10 msec. The next 10 msec could result in $\sim 10^{55}$ ν_e and $\bar{\nu}_e$ with $\varepsilon_\nu \sim 10$ MeV. The total energy involved is $\sim 10^{51}$ ergs. If the neutrinos continue to come out at ~ 10 MeV during deleptonization then another $\sim 10^{58}$ neutrinos could emerge. If the collapse leads to the formation of a black hole then perhaps only the first phase will be observed.

The mass of the detector in metric tons per detected neutrino can then be written as:

$$\frac{M_{\nu_e}}{N_{d,\nu_e}} = 2.2 \times 10^3 \text{ m tons} \left[\frac{D}{10 \text{ kpc}}\right]^2 \left[\frac{\varepsilon_{\nu_e}}{10 \text{ MeV}}\right]^{-1} N^{-1}_{\nu_e,56}$$

and

$$\frac{M_{\bar{\nu}_e}}{N_{d,\bar{\nu}_e}} = 2.1 \times 10^3 \text{ m tons} \left[\frac{D}{10 \text{ kpc}}\right]^2 \left[\frac{\varepsilon_{\bar{\nu}_e}}{10 \text{ MeV}}\right]^{-2} N^{-1}_{\bar{\nu}_e,55}$$

These expressions give a reasonable portrayal of the expected detection rate in the first several tens of milliseconds, the period covered by current calculations. If a simple detection is all that is required and deleptonization produces neutrinos of ~ 10 MeV then a kiloton detector is probably adequate to "see" the 10^{58} neutrinos from a collapse anywhere in the Galaxy. Clearly, however, the case is very marginal for resolving the original burst to compare observations with the theory of core bounce and shock formation. Detection of black hole formation at 10 kpc would be difficult.

GRAVITY WAVES FROM GRAVITATIONAL COLLAPSE

Gravitational radiation, even more surely than neutrinos, emerges from the collapse of a stellar core to form a neutron star. The mechanism for gravity wave production depends on unknown physical factors, however, and detection requires pioneering technology. Despite these difficulties, it is a remarkable testimony to the advance of astrophysics in the last three decades that hopes and efforts have risen hand in hand for detection of gravity waves and of neutrinos.

The contrast is clear between a single pulse of gravitational radiation, associated with a single sudden change in the mass quadrupole moment of the compact object and the continuous train of waves associated with a quadrupole altering periodically with time by reason of vibration or rotation. In either case, the relevant measure of the strength of the gravity waves at the detector is the fractional extension, h, of the length transverse to the direction of the travel of the wave. This quantity is given approximately by

$$h \sim 2GI/c^4Dt^2$$

The reduced quadrupole moment $I \sim 0.1MR^2 \sim 10^{44}$ g cm^{-2}, altering by a large fraction of itself in a time $t \sim 0.1t_{dyn} \sim 10^{-4}$ s at a distance, D, of the order the Galactic center gives $h \sim 10^{-19}$; for the Virgo cluster at about 20 Mpc, $h \sim 10^{-22}$. This estimate is consistent with the detailed calculations of Saenz and Shapiro[11].

Most stars are believed to have an appreciable amount of rotation already before collapse and consequently may rotate very fast indeed after a shrinkage in dimensions of two or three orders of magnitude. At one extreme is a collapse very nearly spherically symmetric, with the rotation of the neutron star at the end so modest that the rotation makes only a small perturbation on the idealized spherical form.

At the other extreme is the "collapse, pursuit, and plunge" scenario envisaged by Ruffini and Wheeler[12]. Here the angular momentum is so great that the neutron fluid assumes a pancake form immediately after collapse. This pancake is unstable and fragments into separate neutron stars that revolve about their common center of gravity, radiating both energy and angular momentum in gravity waves. The pulse generated in such an act of fragmentation should be comparable to, and perhaps even greater than, the pulse generated in the original act of collapse itself. Most estimates of the gravitational radiation to be expected from collapse neglect all but the original pulse. The reason is not lack of interest in those events. It is the difficulty in giving them a detailed hydrodynamic analysis[11].

November 1982 saw the discovery of a pulsar associated with a neutron star spinning around its axis 641 times per second[13]. No object has focussed attention more dramatically on the idea of detecting gravitational waves of such a sharply defined and directly measured frequency. Advance knowledge of the amplitude of the rotating quadrupole will be difficult to acquire despite the accurate period. "Mountains" on the surface of the neutron star less than a centimeter high would suffice to make the Crab pulsar a quite significant source of continuous gravitational radiation, and the new pulsar even more so.

The search for continuous gravitational radiation may tell more about neutron star geology than any present knowledge of that geology can teach about gravitational radiation. No object would seem better suited as a test case than a very fast pulsar such as that picked up by the newly employed fast timing technology at the Arecibo radio telescope.

For the detection of gravitational radiation many ideas have been proposed. Detectors have been built at more than a dozen centers. Descriptions of ideas and devices are available in several books including Misner, Thorne and Wheeler[14] and Smarr[15]. There is always the possibility that some ingenious and novel detector can be invented superior to anything now conceived. An interesting but so far unrealized try in this direction is the proposed device described in CERN reports by Emilio Picasso and Luigi Radicati. Two superconducting cavities each carry an electromagnetic wave. A hole couples them. They respond differently to the gravitational wave by reason of their different orientation to its polarization. In consequence, a little energy is taken from one mode and given to the other. Measurement of this energy transfer is the tool for detection.

Interesting though this and other novel ideas are, present efforts at detection concentrate on two simpler and better known proposals, the Weber bar and the Michelson interferometer.

The bar, with a mass of the order of a ton, has a quadrupole moment and therefore responds to a gravitational wave that travels at any angle to its axis. The expected extension of a bar of length

L = 1 m under the influence of a gravitational wave of amplitude $h = 10^{-22}$ is only of the order of 10^{-20} cm, unbelievably small compared even to the dimensions of an atomic nucleus, and on this account at first sight utterly beyond measurement. However, the quantity one determines experimentally is, in principle, related to an average over all the atoms of the bar, not the position of any individual atom. More concretely, one is concerned with the amplitude of the lowest mode of longitudinal vibration of the bar, typically endowed with a frequency of the order of a 1000 cycle/s. At room temperature this mode carries on the average 4×10^9 quanta; at liquid helium temperature, 4×10^7 quanta. Granted the most favorable phase relation, a sudden gravitational wave with $h \sim 10^{-22}$ will suddenly increase or decrease this number by ~ 100. That is the detection problem. The analysis of the oscillator has to be conducted at the quantum level. By contrast, the gravitational wave, weak though it is, contains such an enormous number of quanta that it can be envisaged for detection purposes in purely classical terms. The frequency of the bar is selected so that it will respond to the expected short pulse, $\sim 10^{-3}$ s, from a collapse event. Current Weber bars operated at room temperature require a wave with $h \sim 10^{-16}$ and operated at 4K, a wave of $h \sim 10^{-17}$, and hence are sufficient to detect only relatively nearby collapse events[16].

The Michelson interferometer, whether on the ground or in space, is conceived as having each mirror, half silvered or fully silvered, stationed on a mass that is free (in space) or effectively free (earth bound but suspended like a pendulum). Ronald Drever of Glasgow and Caltech and his Caltech colleagues have under construction such an interferometer with a forty meter base line. With a mirror reflectivity of R = 0.997 this device would give, Drever estimates, a gravity wave sensitivity of $h \sim 10^{-19}$. His group estimates that improvements are conceivable which would bring this figure to something of the order of 10^{-21}.

CONCLUSIONS

There are many uncertainties involved in the estimate of the detectability of collapse events in the Galaxy. We do not know which supernovae involve collapse. We do not know how often collapse occurs without associated optical display or extended remnant or without either. We do not know how often collapse occurs with negligible magnetic field, with or without a supernova. The details of the neutrino spectrum and time history are also uncertain and subject to revision. We do not know the mass quadrupole moment.

The scientific payoff of the detection of a gravitational collapse event depends on a number of factors. The event must be confirmed. Even then, the simple statement that a confirmed collapse had occurred, while intensely interesting, might be of little benefit if the data is too sparse. The more temporal and spectral information, the more valuable the event. In addition, an event which was susceptible to correlative studies with all the armaments of optical, radio and X-ray astronomy would be of most use.

Neutrino detectors should have thresholds < 10 MeV and time resolution on scales from ~ 0.1 to 1000 msec. In order to confirm an event absolute timing capability for individual detectors is mandatory. Such absolute timing capability would also enable position location through triangulation. If several kiloton detectors were placed about the earth with spacing of order 1000 km the angular resolution would be ~ 1'.Δt (μsec) where Δt is the absolute timing resolution in μsec. Microsecond accuracy would enable a search for an optical counterpart. Timing to ~ 10 nsec could locate a source to an accuracy of order a second of arc.

Discovering an optical counterpart at great distance is not out of the question. Supernovae are estimated to range in peak brightness from -17^m to -20^m depending on the type and distance estimates. The average obscuration in the Galaxy is about 2^m per kpc. Thus a supernova at 10 kpc would appear at m ~ 15 to 18. This is bright enough not only for detection, but for detailed spectral analysis which would determine the type. The actual optical detectability would depend sensitively on the supernova type and the direction in the Galaxy. In some directions the obscuration is very heavy. It is somewhat lower toward the Galactic center, increasing the liklihood of seeing a supernova there.

Type II supernovae are bright and are more likely to be confined to the high obscuration zones of spiral arms. Type I events are brighter and tend to avoid the regions of large obscuration. Simple association of a collapse event with a Type II would not be very useful because it is expected. Lack of collapse would be very surprising but difficult to confirm given the weakness of the signals and the likely obscuration of an optical Type II event. Detection of a Type I and a basic yes/no statement concerning an associable neutrino or gravity waves pulse would be immensely useful in selecting among competing classes of theories.

The chance of an optical, radio, or X-ray correlation (or definite lack thereof) and detailed neutrino or gravity wave temporal and spectral information increases greatly as the distance is reduced. The price, of course, is that the probability of occurrence is lessened approximately as the area of the Galactic plane surveyed.

The low rate of occurrence of supernovae and collapse events in the Galaxy and particularly in the solar neighborhood should not be used as an argument against the construction of appropriate detection devices. Statistics aside, the next event could occur tomorrow. Detection of neutrinos from gravitational collapse should probably not be the first priority, given the low rate, but it would be a tragedy if an event occurred in the near future and the various proton decay experiments were not instrumented in an appropriate manner to detect it.

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